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TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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No. 73.

THE CHOICE OF WING SECTIONS FOR AIRPLANES.

By

Edward P. Warner,
Secretary, Committee on Aerodynamics,
National Advisory Committee for Aeronautics.

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November, 1921.

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THE CHOICE OF WING SECTIONS FOR AIRPLANES.

By

Edward P. Warner
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Ever since the use of wind tunnels first began to give some sort of quantitative data on the comparative performance of aerofoil sections, there has been warm dispute as to the best methods of treating the data thus obtained and as to the most useful criteria for judging the relative merits of various sections. Although scores of such criteria for comparison have been devised and have been advocated with enthusiasm by their inventors, none of them ^{has} have gained universal acceptance, and indeed, it is evident that there can be no single formula for judging the aerodynamic merits of wing sections, even if structural considerations be left out of reckoning entirely. The factors entering are far too numerous, the conditions under which the wings have to work on different types of airplane are too diverse, to permit of the deduction of any single formula which will automatically point the way to the best section in all cases.

The subject of choice of section is by no means a closed one, and despite the impossibility of making a single rule serve, it is quite practicable to deduce in a strictly rational manner a

series of rules and formulas which are capable of being of the greatest use if we but confine ourselves to the consideration of one element of performance at a time.

There are seven such elements of performance which may be taken up in turn, the seven being of different relative importance in different types of airplanes. They are:

- (a) Maximum speed, regardless of minimum
- (b) Maximum speed for given minimum
- (c) Maximum speed range ratio
- (d) Maximum rate of climb
- (e) Maximum absolute ceiling
- (f) Maximum distance non-stop.
- (g) Maximum duration non-stop.

Each of these in turn will be treated separately. (a) and (b) obviously cannot both apply to the same airplane, (a) really being applicable only to racing airplanes which are always to be landed on good fields and where the landing speed can accordingly be increased to abnormally high figures.

A. Choice of Wing Section for Maximum Speed,
Regardless of Minimum.

If the design is to be made with maximum speed as the sole desideratum, obviously the sole requirement is that the drag shall be as small as possible, and since the loading and area can be adjusted so that the airplane will fly at maximum L/D at maximum speed / this is equivalent to choosing for maximum L/D alone. Flying at

maximum speed at maximum L/D, however, entails landing at about 70% of the maximum speed.

In the more common case where a minimum area is initially fixed and the best wing section is to be chosen for that area and a given weight, it is necessary to take account of the power and weight in order to determine the angle of attack at which the airplane flies at maximum speed. At maximum speed

$$C_L V^2 = \frac{W}{S \frac{\rho}{2}} \quad \text{and} \quad (C_D S + C_R) \frac{\rho}{2} V^3 = 550 \eta P$$

where C_R is the coefficient of parasite resistance for the whole airplane, P the engine horsepower, and η the propeller efficiency. V is the speed in ft. per sec. and S is given in sq.ft. If we assume that the parasite resistance is one-half of the total resistance at maximum speed and that the propeller efficiency is 80%, the second equation becomes:

$$2 C_D \frac{\rho}{2} S V^3 = 440 P$$

$$C_D V^3 = \frac{440 P}{\rho S} = \frac{186,000 P}{S}$$

From the equation of lift

$$V = \sqrt{\frac{W}{C_L \frac{\rho}{2} S}}$$

Combining the last two equations, at maximum speed

$$C_D \left(\frac{W}{C_L \frac{\rho}{2} S} \right)^{3/2} = \frac{440 P}{\rho S}$$

$$\frac{C_D^2}{C_L^3} = \frac{(440)^2 \rho P^2 S}{8 W^3} = \frac{57.5 P^2 S}{W^3}$$

a relation which makes it possible to determine approximately the conditions under which a wing will work at maximum speed of a given airplane without the necessity of making detailed performance calculations in advance.

The object is to secure as high a wing efficiency as possible at the maximum speed of flight, or, since resistance is equal to weight divided by L/D , to secure a maximum L/D at a prescribed value of $\frac{C_D^2}{C_L^3}$. The choice of a wing for maximum speed with a given area can therefore be made by plotting L/D against $\frac{C_D^2}{C_L^3}$ for several aerofoils. The best wing section is that which has the largest ordinate at an abscissa equal to $\frac{57.5 P^2 S}{W^3}$, for conventional types. On cantilever-wing designs and others where it is evident that the parasite resistance is less (or more) than one-half the total the constant may be modified accordingly. In cantilever racing monoplanes, for example, 85 may be used instead of 57.5. In comparing sections for a given design, however, the same constant should always be used throughout.

In order to illustrate the use of this method, curves of L/D against $\frac{C_D^2}{C_L^3}$ for a number of sections are plotted in Fig. 1. Taking, for example, the case of a 300 HP pursuit airplane weighing 2500 lbs. and having 240 sq.ft. of wing surface, $\frac{57.5 P^2 S}{W^3} = .0795$. Erecting a vertical on Fig. 1 at this abscissa it is clear that the best results are given by the U.S.A. 16, 17, and 21, and the R.A.F.15.

B. Choice of Wing Section for Maximum Speed
with a Given Minimum.

This case may also be treated in two different ways, depending on whether or not allowance is made for change of weight with changing area. We shall first assume the weight to be fixed independently of wing section and area.

In this case, the wing loading is at once given by the equation:

$$W = C_{Lmax.} \frac{\rho}{2} S V_{min.}^2$$

At maximum speed, just as in case (A):

$$\frac{C_D^2}{C_L^3} = \frac{57.5 P^2 S}{W^3}$$

but S is now a variable and can be eliminated by using the equation of landing speed.

$$\frac{W}{S} = C_{Lmax.} V_{min.}^2 \frac{\rho}{2}$$

$$\frac{C_D^2}{C_L^3} = \frac{57.5 P^2}{C_{Lmax.} V_{min.}^2 W^2 \times \frac{\rho}{2}} = \frac{48400}{C_{Lmax.} V_{min.}^2 W^2 \frac{\rho}{2}}$$

Taking the square root of each side.

$$\frac{\frac{C_L}{C_D}}{\sqrt{\frac{C_{Lmax.}}{C_L}}} = \frac{\frac{W}{P} \times V_{min.}}{220}$$

The wing giving the lowest resistance at maximum speed under these conditions can therefore be found by plotting L/D against

$\frac{L/D}{\sqrt{\frac{C_{Lmax.}}{C_L}}}$ and noting the ordinates at the point where the ab-

scissa is 1/150 of the power loading times the minimum speed, or, alternatively, by plotting against $\frac{C_D^2}{C_L^3}$ and comparing the L/D ratios at the points where the abscissa is equal to $\frac{48,400 P^2}{C_{Lmax.} V_{min.}^2 W^2}$ this abscissa being different for different wing sections.

As an illustration of this method, the same problem may be used as was employed in case (A), specifying a minimum speed of 50 m.p.h. in place of an area of 240 sq.ft. Tabulating the results for the several wings:

Name	$C_{Lmax.}$	$\frac{48,400 P^2}{C_{Lmax.} V_{min.}^2 W^2}$	L/D
U.S.A. 5	1.290	.1005	6.59
15	1.184	.1093	7.38
16	.992	.1304	7.38
17	1.082	.1199	7.81
21	.913	.1419	7.34
27	1.411	.0918	7.03
U.S.A.T.S. 5	1.512	.0858	6.81
10	1.732	.0747	6.46
13	1.294	.1003	6.34
R.A.F. 6	1.200	.1080	6.78
15	1.109	.1161	7.40
Göttingen 387	1.640	.0792	6.93

The comparisons so far, based on the assumption that the wing drag is a fixed proportion of the total, indicate that the ratio of effective parasite resistance surface (equivalent flat plate area) to wing area is larger in the case of a thick wing with a large drag coefficient than in the case of a thin wing. This conclusion is correct in respect of the fuselage, the resistance of which decreases proportionately less rapidly than does

the wing area when the latter is changed. The resistance of the interplane bracing, however, is generally a much smaller fraction of the wing drag for thick wings with high drag coefficients than for thin sections. The method employed is, on the whole, unfair to the thick sections. A possible alternative is to use $\frac{C_L}{C_D + .04}$ instead of L/D , as a criterion of efficiency, assuming that the parasite resistance for the whole airplane is $.04 \frac{\rho}{2} S V^2$ in all cases, A being the wing area. It can readily be shown that, on this assumption, $\frac{(C_D + .04)^2}{C_L^3} = \frac{230 P^2 S}{W^3}$, $\frac{C_L}{C_D + .04}$ must therefore be plotted against $\frac{(C_D + .04)^2}{C_L^3}$. This has been done for 12 sections in Fig. 2. In order to avoid confusion among the large number of curves, the R.A.F.6 and 15 have been indicated by points only. On the whole, this method of comparison is fairer than that previously described, especially where sections of widely varying thickness are to be compared. It will be noted, however, that the conclusions drawn from Figs. 1 and 2 are not very different, the relative rank of the sections being much the same except that the thick sections show up better by the method of Fig. 2.

As a specific example, the problem just solved by the first method will be treated by using Fig. 2. The numerical value of the abscissa for each section must then be just four times as high as in the first case, and the corresponding efficiencies are as tabulated below.

Section		$\frac{C_L}{C_D + .04}$
U.S.A.	5	3.25
	15	3.33
	13	3.25
	17	3.29
	21	3.09
	27	3.39
U.S.A.T.S.	5	3.46
	10	3.32
	13	3.16
R.A.F.	3	3.23
	15	3.33
Göttingen	387	3.45

The six best sections by the two methods, arranged in order of merit, are:

Rank	By Fig. 1	By Fig. 2
1	U.S.A. 17	U.S.A.T.S. 5
2	R.A.F. 15	Göttingen 387
3	U.S.A. 16	U.S.A. 27
4	U.S.A. 15	R.A.F. 15
5	U.S.A. 21	U.S.A. 15
6	U.S.A. 27	U.S.A.T.S. 10

Turning now to the derivation of a formula to take account of the change in weight with varying area, it can be shown that the total weight of an airplane is approximately proportional to $S^{1/6}$, other things being equal, where A is the wing area.

The equation of maximum speed then becomes:

$$\frac{C_D^2}{C_L^3} = \frac{57.5 P^2 S}{(K S^{1/6})^3} = \frac{57.5 P^2 \sqrt{S}}{K^3}$$

If W_0 and S_0 represent the initial weight and the corresponding area for some particular wing section,

$$W_0 = K S_0^{1/6}$$

$$K^3 = \frac{W_0^3}{\sqrt{S_0}} = W_0^{5/2} V_{\min.} \sqrt{\frac{C_{L\max.} \rho}{2}}$$

Since the $C_{L\max.}$ in this last equation relates to a particular wing section, that for which W_0 and S_0 are taken, it must be a constant. Taking 1.3 as an average value of $C_{L\max.}$ and substituting, and then combining the last two equations,

$$K^3 = .039 W_0^{5/2} V_{\min.}$$

$$\frac{C_D^2}{C_L^3} = \frac{57.5 P^2 \sqrt{\frac{2 W}{C_{L\max.} \rho}}}{.039 V_{\min.}^2 W_0^{5/2}} = \frac{42,800 P^2 \sqrt{\frac{W}{C_{L\max.}}}}{V_{\min.}^2 W_0^{5/2}}$$

Neglecting the small difference between $\sqrt{W}$ and $\sqrt{W_0}$ in any particular case, this becomes:

$$\frac{C_D^2}{C_L^3} = \frac{42,800 P^2}{\sqrt{C_{L\max.}} V_{\min.}^2 W_0^2}$$

In this case, since the weight is not a constant, it does not suffice to compare values of L/D .

Resistance at maximum speed

$$\frac{W}{L/D} = \frac{K S^{1/6}}{L/D} = \frac{K W^{1/6}}{L/D \pi V_{\min.}^{1/3} C_{L\max.}^{1/6} \left(\frac{\rho}{2}\right)^{1/6}}$$

Neglecting the small changes which would occur in $W^{1/6}$, it becomes apparent that the minimum total resistance for a given weight power and minimum speed is given by the wing which has the largest

value of $L/D \times \sqrt[6]{C_{L_{max.}}}$ at the point where

$$\frac{C_D^2}{C_L^3} = \frac{42,800 P^2}{\sqrt{C_{L_{max.}}} V_{min.}^2 W_0^2}$$

Since $C_{L_{max.}}$ has a single definite value for each section, this comparison, like the preceding one, can be made by plotting L/D against $\frac{C_D^2}{C_L^3}$, the ordinate being read off at the appropriate abscissa for each wing section and each ordinate so determined being multiplied by $\sqrt[6]{C_{L_{max.}}}$.

Treating the same illustrative problem as before, tabulating the various quantities involved for each section, a new order of merit is obtained.

Section	$C_{L_{max.}}$	$\sqrt{C_{L_{max.}}}$	$\sqrt[6]{C_{L_{max.}}}$
U.S.A. 5	1.290	1.133	1.042
15	1.164	1.088	1.028
16	.992	.996	.999
17	1.082	1.040	1.013
21	.913	.955	.965
27	1.411	1.188	1.059
U.S.A.T.S. 5	1.512	1.230	1.072
10	1.732	1.317	1.096
13	1.294	1.138	1.044
R.A.F. 6	1.200	1.095	1.031
15	1.109	1.053	1.017
Göttingen 387	1.640	1.280	1.086

Section	$\sqrt{\frac{42,800 P^2}{C_{L_{max}} V_{min}^2 W_0}}$	L/D	$L/D \times \sqrt{C_{L_{max}}}$
U.S.A. 5	.100	6.60	6.87
15	.104	7.37	7.58
16	.114	7.67	7.66
17	.109	8.01	8.11
21	.119	7.73	7.61
27	.095	6.93	7.34
U.S.A.T.S. 5	.092	6.65	7.13
10	.086	6.14	6.73
13	.100	6.34	6.62
R.A.F. 6	.104	6.87	7.08
15	.108	7.60	7.73
Göttingen 387	.089	6.66	7.23

The relative merit of the sections as shown by this table is almost identical with that determined from the analysis with changes of weight ignored. In all practical problems the simpler type of treatment, by which the relative efficiencies are read off directly from the L/D chart, will suffice.

This case, like the one in which the weight is assumed constant, can readily be treated on the assumption of a fixed parasite resistance coefficient in place of that of a fixed ratio of parasite resistance to wing drag. As has just been shown, however, the difference between the results with the two assumptions is so small that it would hardly be worth while carrying through another example.

These methods of comparison, while accurate, are rather complex, and it is worth while seeking a somewhat simpler device. Such a device can readily be obtained for the case of landing speed and constant weight. Writing the equation,

$$D = \frac{W}{L/D} = \frac{W}{\frac{C_{L_{max.}}}{C_D} \times \frac{C_L}{C_{L_{max.}}}} = \frac{W}{C_D} \times \left(\frac{V}{V_{min.}} \right)^2$$

Since all airplanes having a speed ratio of from 1.8 to 2.8 (all, in short, except a few with an exceptionally high power loading) fly when at maximum speed at an angle of attack close to that of minimum drag, and since the drag coefficient curve is very flat in the neighborhood of its minimum ordinate, little error will result from inserting $C_{D_{min.}}$ for C_D in the above equation. In general, therefore, the wing best suited for maximum speed is that for which the value of $\frac{C_{L_{max.}}}{C_{D_{min.}}}$ is highest. The value of this ratio has been tabulated below for a few wing sections, all tested at the Massachusetts Institute of Technology wind tunnel under the same conditions.

Aerofoil	$\frac{C_{L_{max.}}}{C_{D_{min.}}}$
U.S.A. 5	44.9
15	53.0
16	52.9
17	60.4
21	52.7
27	50.7
U.S.A.T.S. 5	43.6
10	36.8
13	35.0
Göttingen 387	44.5
R.A.F. 6	42.0
15	57.8

C. Choice of Section for Maximum Speed Range Ratio.

Writing, as in (b), the equations of maximum and minimum speed, with appropriate allowance for dependence of weight on area:

$$C'D v_{\max.}^3 = \frac{220 P}{\frac{\rho}{2} S}$$

$$v_{\max.} = \sqrt[6]{\frac{48,400 P^2}{(\frac{\rho}{2})^2 S^2 C'D^2}}$$

$$W = C_{L\max.} \frac{\rho}{2} S v_{\min.}^2$$

$$v_{\min.} = \sqrt[6]{\frac{W^3}{C_{L\max.}^3 S^3 (\frac{\rho}{2})^3}}$$

$$\frac{v_{\max.}}{v_{\min.}} = \sqrt[6]{\frac{48,400 P^2 S C_{L\max.}^3 \frac{\rho}{2}}{W^3 C'D^2}} = \sqrt[6]{\frac{57.5 P^2 S C_{L\max.}^3}{W^3 C'D^2}}$$

Substituting $K S^{1/6}$ for W ,

$$\frac{v_{\max.}}{v_{\min.}} = \sqrt[6]{\frac{57.5 P^2 C_{L\max.}^3}{K^3 C'D^2}} \times \sqrt[12]{S}$$

It therefore appears, since S is a variable quite independent of all the rest, that speed range ratio can be increased without limit by a sufficient increase in wing area. The gain is, however, very slow, an increase of 50% in area raising the ratio by only 4%. The comparison of wing sections can be made most reasonably by assuming a constant area. If this be done, the best section is that giving the highest value of $\frac{C_{L\max.}^3}{C'D^2}$, where $C'D$ is

the drag coefficient at an angle so chosen that

$$\sqrt[6]{\frac{57.5 P^2 C_{Lmax.}^3}{K^3 C'D^2}} \times \sqrt[12]{S} = \sqrt{\frac{C_{Lmax.}}{C'_L}}$$

$$\frac{\sqrt[3]{C'D}}{\sqrt{C'_L}} = \sqrt[6]{\frac{57.5 P^2}{K^3}} \times \sqrt[12]{S}$$

$$\frac{C'D^2}{C'_L^3} = \frac{57.5 P^2 S^{1/2}}{K^3}$$

Since S is assumed to have been initially fixed, both S and W can be considered as constants, and

$$K^3 = \frac{W^3}{\sqrt{S}}$$

$$\therefore \frac{C'D^2}{C'_L^3} = \frac{57.5 P^2 S}{W^3}$$

the same condition as was initially found to define maximum horizontal speed in case (a). The original plot of L/D against $\frac{C_D^2}{C_L^3}$ can therefore be used, the angle of attack which gives the appropriate value of $\frac{C_D^2}{C_L^3}$ being determined and the cube of the maximum lift coefficient of the wing under consideration being divided by the square of the drag coefficient at the angle thus determined. The best section is the one showing the largest value for this quantity.

Returning to the same problem as before,

$$\frac{57.5 P^2 S}{W^3} = .0795$$

Section	$C_{L_{max.}}$	$(L/D)'$	$C'D$	$\frac{C^3_{L_{max.}}}{C'D^2}$
U.S.A. 5	1.290	6.59	.0443	1616
15	1.184	7.28	.0325	1306
16	.992	7.38	.0314	792
17	1.082	7.81	.0263	1236
21	.913	7.34	.0318	616
27	1.411	7.03	.0365	2039
U.S.A.T.S. 5	1.512	6.81	.0400	2219
10	1.732	6.46	.0467	2858
13	1.294	6.34	.0494	1121
R.A.F. 6	1.200	6.78	.0404	1090
15	1.109	7.40	.0310	1133
Göttingen 387	1.640	6.93	.0380	2956

The advantage in this comparison clearly rests very strongly with the thick, high-lift sections.

If the assumption of constant parasite resistance is made, the problem becomes to find the wing with the highest value of $\frac{C^3_{L_{max.}}}{(C'D + .04)^2}$ where $C'D$ is determined by the condition:

$$\frac{(C'D + .04)^2}{C'L^3} = \frac{230 P^2 S}{W^3}$$

This solution would show the thick wings, all of which have high minimum drag coefficients, in an even more favorable light than the preceding one.

D. Choice of Section for Maximum Rate of Climb.

Although the air speed for best climb is not in general identical with that for minimum power required, nevertheless the two are close enough together so that the problem of securing a maximum rate of climb reduces itself essentially, provided that the weight remains constant, to the problem of cutting down to as low a value as possible the minimum power required for horizontal

flight.

The rate of climb can be expressed by the equation:

$$C = \frac{\eta P}{W} - \frac{(C_D S + C_R) \frac{\rho}{2} V^3}{W}$$

where c is the rate of climb.

If it be assumed that the parasite resistance is 40% of the total resistance under conditions of best climb and that the propeller efficiency is 75%, the equation becomes:

$$C = \frac{412 P}{W} - \frac{5/3 C_D S V^3}{C_L S V^2} = \frac{412 P}{W} - \frac{5 V}{3 L/D}$$

In this case, as in that of maximum speed, the area may be treated either as a constant or as a variable. If S be taken as fixed at the same value for all wings W is also a constant, and the problem becomes simply one of making $\frac{L/D}{V}$ as large as possible. With W and S constant,

$$V = \sqrt{\frac{W}{\frac{\rho}{2} S}} \times \sqrt{\frac{1}{C_L}}$$

$$\frac{L/D}{V} \propto \frac{L}{D} \times \sqrt{L_C}$$

and the maximum value of this function can be found by plotting it directly against angle.

For a more accurate analysis the equation given above is not quite satisfactory, as it leads to emphasizing the properties which the wing possesses at an angle smaller than that of best climb. It is better, instead of taking the parasite resistance

coefficient as a certain proportion of the wing drag coefficient, to take it as an additive constant. The average value of this constant as already noted, is .04 A. Therefore

$$C = \frac{412 P}{W} - \frac{(C_D + .04) \frac{\rho}{2} S V^3}{C_L \frac{\rho}{2} S V^2} = \frac{412 P}{W} - \frac{(C_D + .04) V}{C_L}$$

If the weight and area are constant, therefore, the best wing for climb is that which has the largest maximum value of $\frac{C_L^{3/2}}{C_D + .04}$, a function the maximum value of which for each section must be found by computing it directly for a few angles and plotting against angle.

If the area is allowed to vary without restriction, the weight being assumed to depend on area in the same manner as heretofore, the equation becomes:

$$C = \frac{412 P}{K S^{1/6}} - \frac{(C_D + .04) \sqrt{\frac{K S^{1/6}}{S}} \sqrt{\frac{1}{\rho/2 C_L}}}{C_L} =$$

$$\frac{412 P}{K S^{1/6}} - \frac{(C_D + .04) \sqrt{K}}{S^{5/12} C_L^{3/2} \left(\frac{\rho}{2}\right)^{1/2}}$$

$$\frac{dC}{dS} = - \frac{69 P}{K S^{7/6}} + \frac{5 (C_D + .04) \sqrt{K}}{12 S^{17/12} C_L^{3/2} \left(\frac{\rho}{2}\right)^{1/2}}$$

considering C_D and C_L as constants. For maximum climb

$$\frac{69 P}{K S^{7/6}} = \frac{5 (C_D + .04) \sqrt{K}}{12 S^{17/12} C_L^{3/2} \left(\frac{\rho}{2}\right)^{1/2}}$$

$$\frac{5.61 P}{K S^{7/6}} = \frac{(C_D + .04) \sqrt{K}}{S^{17/12} C_L^{3/2}}$$

$$\frac{5.61 P S^{1/4}}{K^{3/2}} = \frac{C_D + .04}{C_L^{3/2}}$$

Substituting the value thus determined for $S^{1/4}$,

$$C = \frac{1}{S^{1/6}} \left[\frac{412 P}{K} - \frac{(C_D + .04) \sqrt{K}}{C_L^{3/2} \left(\frac{\rho}{2}\right)^{1/2}} \times \frac{5.61 P C_L^{3/2}}{K^{3/2} (C_D + .04)} \right] =$$

$$\frac{1}{S^{1/6}} \left[\frac{412 P}{K} - \frac{166 P}{K} \right] = \frac{246 P}{K S^{1/6}} = \frac{246 P}{W}$$

$$S^{1/6} = \left[\frac{(C_D + .04) K^{3/2}}{5.61 P C_L^{3/2}} \right]^{2/3}$$

$$C = \frac{246 P}{K} \left[\frac{5.61 P C_L^{3/2}}{(C_D + .04) K^{3/2}} \right]^{2/3} = \frac{777 P^{5/3} C_L}{(C_D + .04)^{2/3} K^2} =$$

$$\frac{777 P^{5/3}}{K^2} \left(\frac{C_L^{3/2}}{C_D + .04} \right)^{2/3}$$

The values of K for a few modern airplanes are tabulated below

	K
D.H. 4	1300
Vought V.E. 7	820
Curtiss JN4D	760
Fokker D7	800
Martin Bomber	3200
Handley-Page (4-engined)	6550
Thomas-Morse MB3	830

further
As a basis for comparison, the maximum values of $\frac{C_L^{3/2}}{C_D + .04}$ have been calculated for several good aerofoil sections, all of which have been tested under the same conditions at the Massachusetts Institute of Technology, and are tabulated at the end of this section.

As illustrations, the theoretical best area will be calculated

ed for airplanes of the type of the Vought and Martin bomber, assuming the U.S.A.15 wing to be used in both cases.

$$S = \left[\frac{\text{Vought} (C_D + .04) K^{3/2}}{5.61 P C_L^{3/2}} \right]^4$$

$$\frac{C_D + .04}{C_L^{3/2}} = \frac{1}{8.65} = .116 \text{ (See table).}$$

$$K = 820$$

$$P = 180$$

$$A = (2.68)^4 = 51 \text{ ft.}^2$$

Martin

$$K = 3200$$

$$P = 800$$

$$A = \left(.116 \times \frac{3200 \times 56.6}{5.61 \times 800} \right)^4 = (4.64)^4 = 460 \text{ ft.}^2$$

It will be observed that these optimum areas are far below those actually employed, and we conclude that the rate of climb at ground level could be materially improved for most present-day airplanes by decreasing the area. This, however, would be very injurious to the ceiling and to the rate of climb at high altitudes. If the best area is to be found for rate of climb at 10,000 ft., for example, instead of at sea level, the engine power being reduced by 28% and the power required for flight at a given angle of attack being increased by 18%, the equating to zero of the derivative of the rate of climb gives:

$$.72 \times \frac{69 P}{KS^{7/6}} = \frac{1.18 \times 5 (C_D + .04) \sqrt{K}}{13 S^{17/12} C_L^{3/2} \left(\frac{P}{2}\right)^{1/2}}$$

$$\frac{3.43 P S^{1/4}}{K^{3/2}} = \frac{C_D + .04}{C_L^{3/2}}$$

The best area for rate of climb at 10,000 ft. is therefore 6.55 times as large as for rate of climb at sea level. This works out at 330 sq.ft. for the Vought, 3030 sq.ft. for the Martin, the first figure slightly larger, the second much larger than the areas actually employed. The optimum area for any given type of airplane runs up very rapidly as higher altitudes are considered, approaching infinity as the ceiling is approached.

Whatever altitude may be taken as a basis, however, and whatever may be the type of airplane, the aerofoil section which gives the best rate of climb with a free choice of area is the section which gives the largest maximum value of $\frac{C_L^{3/2}}{C_D + m}$, and .04 is a good average value for m .

It should be remembered, of course, that this discussion has been based on an implicit assumption that the propeller, as well as the wing section, is to be chosen for climb alone. If this were the case, most airplanes would climb best at an angle of attack of about 10° , under which conditions $\frac{C_L^{3/2}}{C_D + .04}$ reaches its maximum. In service airplanes, however, the propeller is usually designed for best performance in the neighborhood of the maximum horizontal speed, and the propeller efficiency falls off rapidly

with decreasing speed of flight. The best climb in actual practice is therefore obtained at an angle of attack of about 6° at sea level in most cases, and the wing performance in terms of power consumed under those conditions is approximated to by the maximum value of $\frac{C_L^{3/2}}{C_D}$, as already noted.

There are, then, two excellent criteria of rate of climb, the maximum value of $\frac{C_L^{3/2}}{C_D + .04}$ and that of $\frac{C_L^{3/2}}{C_D}$. The first is especially suited for judging performance at high altitudes and for use in connection with airplanes which have a very small speed range ratio, or which, like some commercial and bombing types, are designed for economy rather than for extreme speed and ceiling and which accordingly employ propellers giving their best efficiency at a speed considerably below the maximum.

To facilitate comparison, both criteria have been tabulated together for a dozen good wing sections.

	$\frac{C_L^{3/2}}{C_D + .04}$	$\frac{C_L^{3/2}}{C_D}$
U.S.A. 5	9.05	13.31
15	8.66	13.13
16	7.75	12.38
17	8.29	12.51
21	7.03	12.26
27	9.05	13.38
U.S.A.T.S. 5	8.85	11.78
10	9.31	12.59
13	8.79	11.92
R.A.F. 6	8.49	12.69
15	8.10	12.06
Göttingen 387	9.21	12.32

If the wing area, instead of being varied at will, is kept constant, it is obvious that maximum climb reduces simply to a question of minimum power consumption, and that the same two criteria hold as in the previous case, with the same general rules governing the choice between them. The numerical relations between the climbing speeds with any particular pair of aerofoils, are, however, very different for the fixed and for the variable area.

The case of best climb for a given landing speed is rather complex to treat, and arises so rarely that discussion is hardly worth while.

E. Choice of Wing Section for Maximum Absolute Ceiling.

In this case, as in the last one, it will be assumed that the best possible propeller for the particular characteristic under examination is employed.

It can readily be shown that the best propeller for ceiling is one which causes the curves of power available and power required to meet at the ceiling in such a way that their common tangent is horizontal, as shown in Fig. 3. Furthermore, it appears that if such a propeller is used, the ceiling depends only on the ratio of maximum power available at sea level to minimum power required at sea level.

Since power available is dependent neither on wing characteristics nor on weight, the requirement for maximum ceiling reduces itself to a simple matter of keeping the minimum power re-

quired for horizontal flight as small as possible. This condition has already been treated for the case of a fixed wing area, under (D), where it was shown to be roughly true that minimum power is required with the section which gives the largest value of $\frac{C_L^{3/2}}{C_D + .04}$.

It is impossible to find a "best area" for ceiling, as was done for rate of climb, as it is theoretically possible to increase the ceiling without limit by sufficiently increasing the area. A practical case which does arise frequently, however, is that in which it is desired to obtain maximum ceiling in conjunction with a given maximum speed. In order to secure the same maximum speed on the same type of airplane with several different wing sections, the areas must obviously be different. To determine the area for a given maximum speed with each section, the analysis undertaken in the early part of the paper must be carried further.

It has already been shown that

$$\frac{C_D^2}{C_L^3} = \frac{57.5 P^2 \sqrt{S}}{K^3}$$

and

$$\frac{L}{D} = \frac{W V}{220 P} = \frac{K V S^{1/6}}{220 P}$$

if the weight be assumed proportional to the sixth root of the area. Therefore,

$$\frac{\frac{C_D^2}{C_L^3}}{\left(\frac{C_L}{C_D}\right)^3} = \frac{6110 (10 P)^5}{K^3 V^3}$$

If a value of V be chosen and K be assumed and the power is known it is now possible to solve for

$$\frac{\frac{C_D^2}{C_L^3}}{\left(\frac{C_L}{C_D}\right)^3}$$

and the point at which the prescribed value occurs for any particular section can readily be determined by trial from the curve of $\frac{L}{D}$ against $\frac{C_D^2}{C_L}$, dividing the abscissa by the cube of the ordinate. This point being located, the abscissa is multiplied by the square of the ordinate to find the lift coefficient and so the area.

$$\frac{C_D^2}{C_L^3} \times \left(\frac{C_L}{C_D}\right)^2 = \frac{1}{C_L} = \frac{\frac{\rho}{2} S V^2}{W} = \frac{\frac{\rho}{2} S V^2}{K S^{1/6}} = \frac{\frac{\rho}{2} S^{5/6} V^2}{K}$$

$$S = \left[\frac{K}{\frac{\rho}{2} V^2} \times \frac{C_D^2}{C_L} \times \left(\frac{L}{D}\right)^3 \right]^{6/5}$$

The maximum area permitting of a given maximum speed is thus determined.

The minimum HP required is:

$$P_{\min.} = \frac{(C_D + .04) \frac{\rho}{2} S V^3}{550} = \frac{(C_D + .04) S V^{3/2} \frac{\rho}{2}}{550 S^{3/2} C_L^{3/2} \left(\frac{\rho}{2}\right)^{3/2}} = \frac{(C_D + .04) K^{3/2}}{18.9 C_L^{3/2} S^{1/4}}$$

The section giving the best ceiling with a given maximum speed is

therefore the one giving the lowest minimum value of $\frac{C_D + .04}{C_L^{3/2} S^{1/4}}$, where A is calculated in accordance with the procedure described in the preceding paragraph.

To illustrate the use of this method, another problem will be worked. Given a pursuit airplane with a 300 HP engine and K equal to 1000 (the value corresponding to a weight of 2500 lbs. and an area of 240 sq.ft.), the problem is to choose the wing section giving the highest ceiling in conjunction with an air speed at sea level of 140 m.p.h., or 205 ft. per sec. This problem, like the preceding ones, can best be solved by systematic tabulation.

$$\frac{1940 (10 P)^5}{K^5 V^3} = \frac{6110 (3000)^5}{1000^5 \times 205^3} = .000172$$

Section	$\frac{C_D^2}{C_L^3}$	$\frac{L}{D}$	A	Min. $\frac{C_D + .04}{C_L^{3/2} S^{1/4}}$	Rank
U.S.A. 5	.0722	7.53	198	.0294	4
15	.0840	7.93	268	.0285	2
16	.0924	8.18	325	.0305	
17	.0962	8.21	344	.0280	1
21	.0957	8.20	340	.0331	
27	.0743	7.61	210	.0290	3
U.S.A.T.S. 5	.0676	7.37	174	.0311	
10	.0595	7.03	133	.0315	
13	.0664	7.31	166	.0317	
R.A.F. 6	.0750	7.65	215	.0308	
15	.0891	8.09	303	.0296	5
Göttingen 387	.0674	7.34	171	.0299	6

The thin sections thus appear to have some advantage over the best of the thicker ones, but, as already pointed out, this method hardly gives due weight to the great structural advantages

of the thick section. A somewhat fairer comparison between sections of widely different thickness can be made by assuming a constant resistance coefficient of .0001 throughout. It is obvious from what has gone before that if this be done the limiting condition at maximum speed becomes:

$$\frac{62,100 (10 P)^5}{K^6 V^3} = \frac{(C_D + .04)^2}{\left(\frac{C_L}{C_D + .04}\right)^3}$$

and the area equation is:

$$S = \left[\frac{K}{V^2} \times \frac{(C_D + .04)^2}{C_L^3} \times \left(\frac{C_L}{C_D + .04} \right)^2 \right]^{6/5}$$

while the final criterion of merit is the minimum value of

$$\frac{C_D + .04}{C_L^{3/2} S^{1/4}}, \text{ as before.}$$

The illustrative problem may now be attacked anew by this method.

$$\frac{62,100 (10 P)^5}{K^6 V^3} = .0055.$$

Section	$\frac{(C_D + .04)^2}{C_L^3}$	$\frac{C_L}{C_D + .04}$	A	Min. $\frac{C_D + .04}{C_L^{3/2} S^{1/4}}$	Rank
U.S.A. 5	.281	3.71	185	.0300 ...	2
15	.302	3.79	212	.0303 ...	4
16	.308	3.81	220	.0336	
17	.304	3.80	215	.0315	
21	.302	3.79	213	.0372	
27	.286	3.72	190	.0298 ...	1
U.S.A.T.S. 5	.275	3.68	175	.0311 ...	6
10	.250	3.56	145	.0310 ...	5
13	.266	3.64	166	.0317	
R.A.F. 6	.286	3.72	190	.0318	
15	.305	3.81	217	.0321	
Göttingen 387	.268	3.64	167	.0301 ...	3

The order of merit of the best sections is somewhat different from that determined by the first method, although four of the sections appear among the best six in both tabulations. Some judgment must be used in deciding which table to employ, the choice depending on the nature of the structure and the extent to which parasite resistance can be reduced when the wing section is thickened. Furthermore, thin sections with high minimum drag coefficients, such as the U.S.A.5, should be viewed with suspicion when they give very much better relative results by the second method than by the first, as the proportional reduction of total parasite resistance in changing from the U.S.A.15 to the U.S.A.5, for example, would not be as great as the reduction of area.

The type of problem just solved is the most important one possible in a high altitude pursuit airplane. It appears clear that among the twelve sections considered, the best suited for use on such an airplane from the standpoint of performance alone

is the U.S.A.27, with the Cöttingen 387, U.S.A.15, and U.S.A.5 following in that order.

F. Choice of Wing Section for Maximum Radius of Action.

Again supposing the propeller to be working at its maximum efficiency, problem of radius of action is one merely of minimum resistance, or, in other words, of maximum L/D of the airplane. Since for a fixed efficiency of propeller and power plant, a definite proportion of the heat units in the fuel reappear as useful work of propulsion, and since this useful work is equal to the product of resistance by distance flown, the distance possible with a given weight of fuel is obviously inversely proportional to the resistance.

If the coefficient of total resistance in terms of wing area be taken, as before, as $C_D + .04$, (this assumes the equivalent flat-plate area to be 3.1% of the wing area), the total resistance of the airplane is:

$$\frac{\frac{W}{C_L}}{C_D + .04}$$

This is obviously a minimum for minimum area, and the radius of action would therefore be increased by clipping the wings if the amount of fuel carried remained constant. If, on the other hand, the proportion of fuel weight to total weight is constant the radius of action is independent of weight and area. In actual fact the proportion of weight carried as fuel increases with in-

creasing area, and the radius of action is therefore actually largest when the area is largest. Whatever area may be selected, however, the best wing section is the one giving the largest maximum value of $\frac{C_L}{C_D + .04}$ a function which is tabulated below for a number of sections.

It is obvious that the requirements for covering a given distance with a minimum fuel expense are the same as those for covering the maximum distance with a given weight of fuel, and the same criterion therefore serves as a measure of commercial economy as well as of radius of action.

Name	:	Max. $\frac{C_L}{C_D + .04}$
U.S.A. 5	:	8.74
15	:	8.55
16	:	8.26
17	:	8.45
21	:	8.06
27	:	8.78
U.S.A.T.S. 5	:	8.02
10	:	8.37
13	:	8.09
R.A.F. 6	:	8.41
15	:	8.11
Göttingen 387	:	8.38

G. Choice of a Wing Section for Maximum Duration.

Flights of very long duration are seldom of great practical value in themselves, but the making of a new duration record always attracts enough interest and serves as a spectacular enough demonstration so that the requirements for securing maximum duration of flight must not be overlooked entirely.

If it be again assumed that the propeller and power plant efficiency are constant the securing of a maximum duration is simply a matter of cutting down the minimum power required, as the weight of fuel which the airplane can lift represents a certain definite number of horsepower hours, and any reduction in the power makes possible an increase of the number of hours. It is too obvious to require proof that the possible duration increases with increase of area, practically the only limit to the possible gains in this way being the drop in propeller efficiency as the speed of flight falls off with increasing area.

It has already been shown that the minimum horsepower required is secured with that wing section which gives the largest maximum value of $\frac{C_L^{3/2}}{C_D + .04}$. The conditions for maximum duration are closely akin to those for maximum climb.

Horsepower

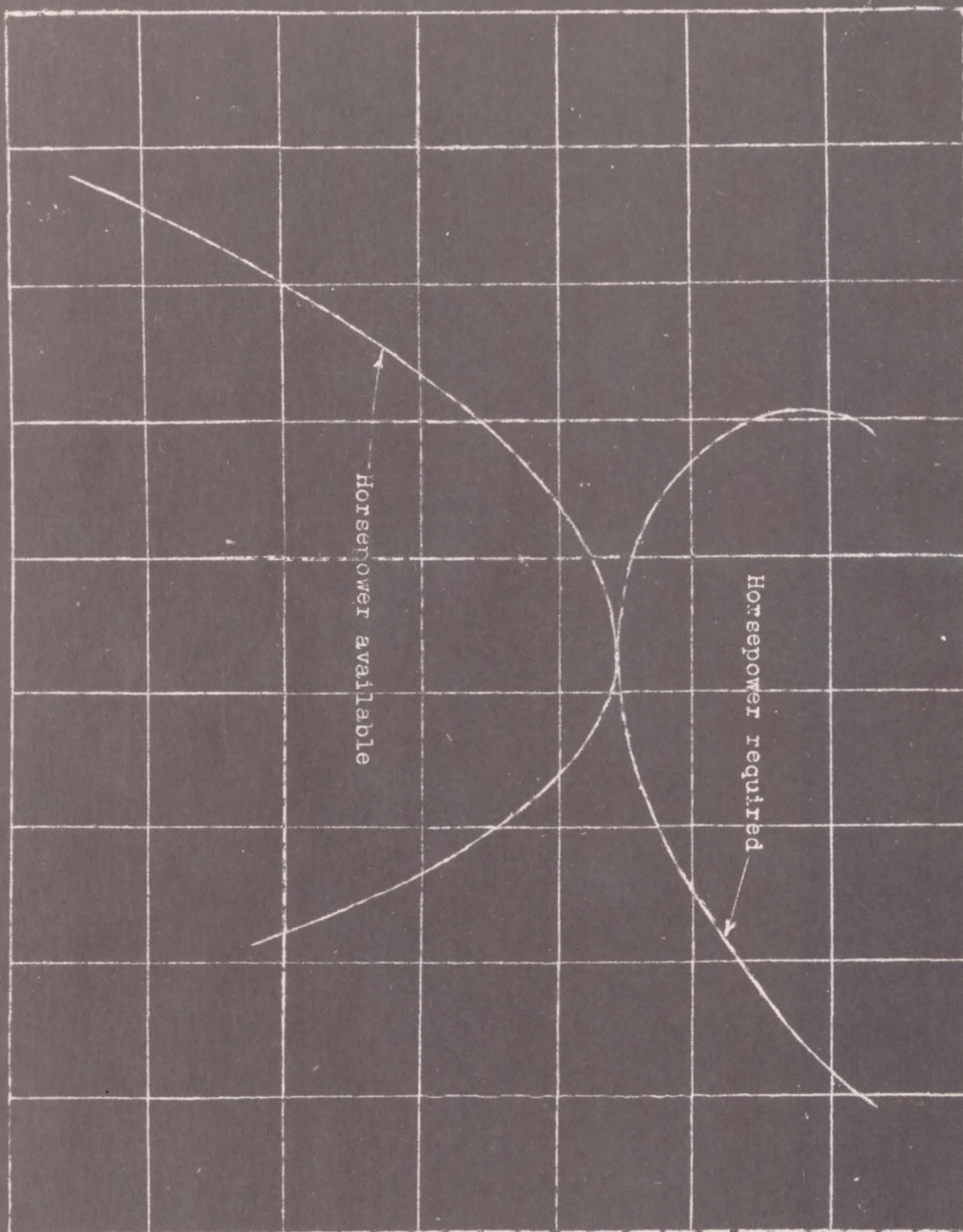
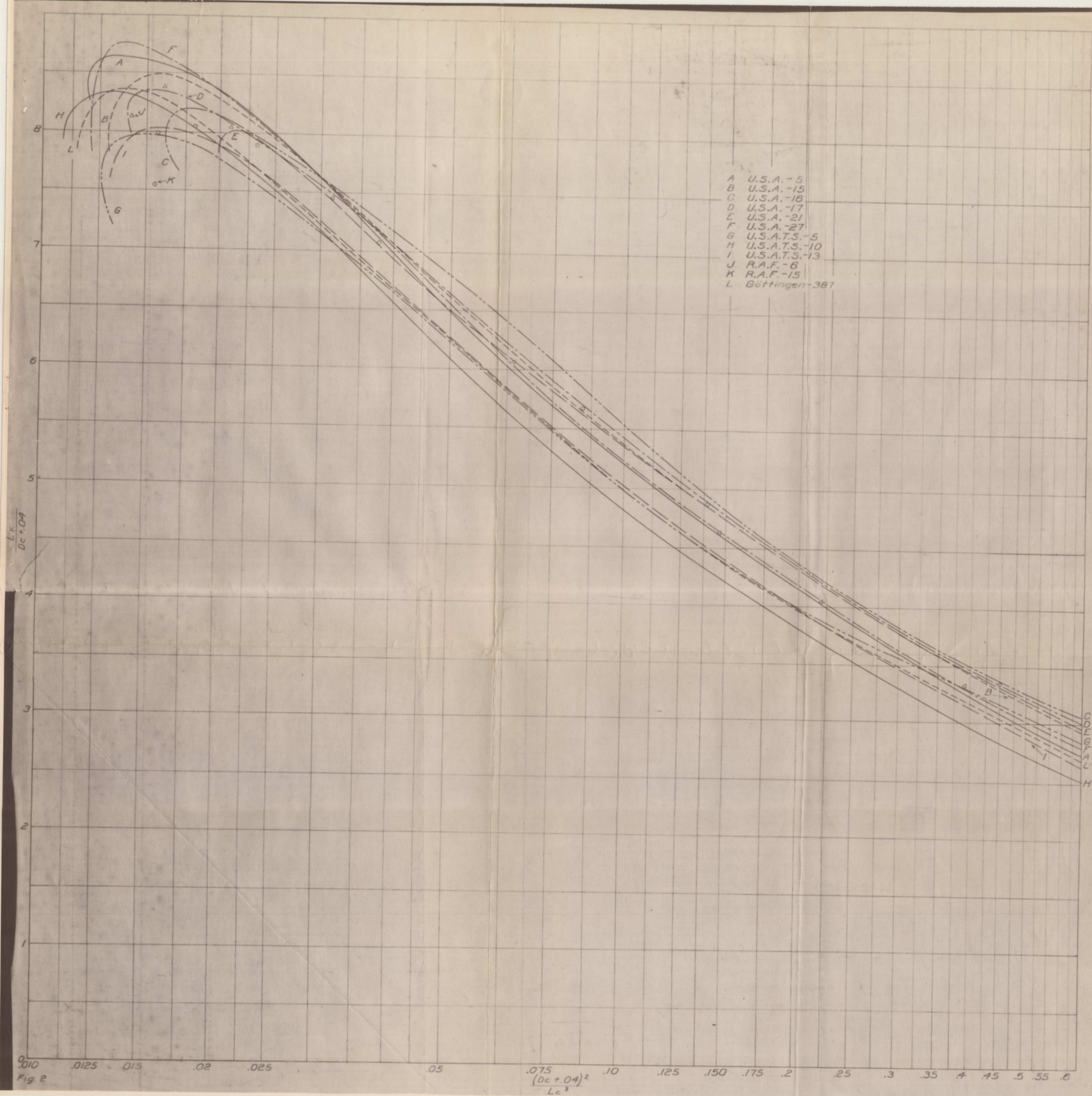


Fig. 3

V



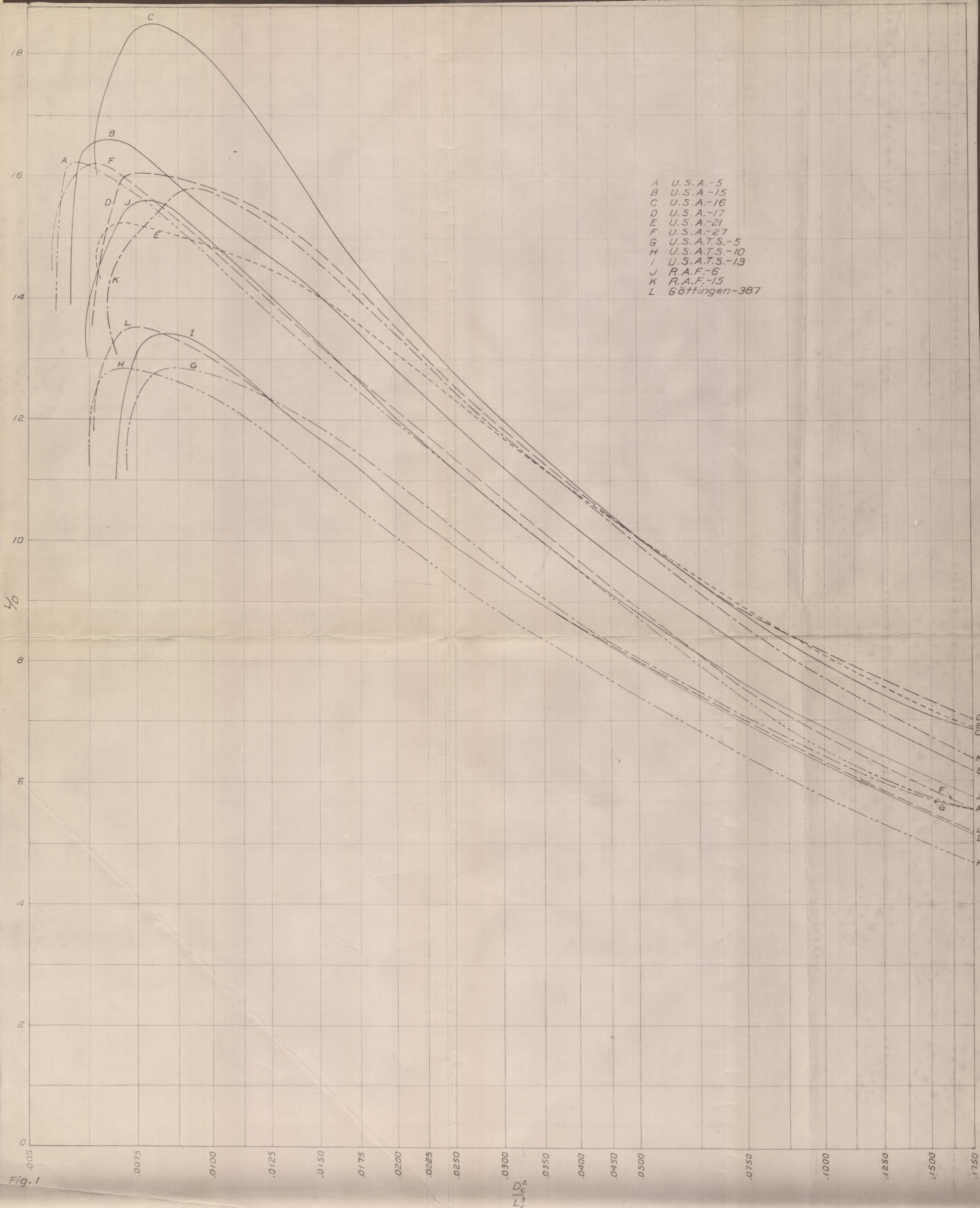


Fig. 1